where

$$x'_{ni} = (\tan\gamma_n)^{-1} \sum_{k=1}^{n-1} s_k \tan\gamma_k \tag{16}$$

$$x'_{nf} = x'_{ni} + s_n \tag{17}$$

$$L_n = R_n / \sin\gamma_n \tag{18}$$

If the first inequality in Eq. (15) holds, the end of the contour has not been reached by the end of the segment, while if the second inequality holds, it has. Thus the total area of the contour for N segments is given by

$$A_T = A_1 + \sum_{n=2}^{N} \left[F_n \right]_{x'_{ni}}^{\mathrm{UL}} \tag{19}$$

Contour Extremity

The end of the contour is of special interest, and it can be determined from the equations already given. On the last segment, $X'_{\max} = (x'_{\max}\ \ 0)^T$ and $x'_{\max} = L_n$. Then, using the transformations in Eqs. (8) and (9), one obtains

$$X_{\max} = X''_{\max} + \Delta_n = \Lambda_n^T X'_{\max} + \Delta_n \tag{20}$$

Other Considerations

Landing problems have not been considered in the previous analysis. However, it is obvious that such problems can be solved by suitable transformations to an equivalent takeoff problem. The details are discussed in Ref. 4.

Implementation of the preceding equations can be accomplished readily on a small hand-held programmable calculator so as to rapidly determine the quantities of interest, namely, the contour area, the contour, and its extremity for arbitrary flight paths. The repetitive nature of the calculations for successive segments is well-suited to such implementation. A program has been devised for the Hewlett-Packard HP-67, the details of which are given in Ref. 4. The inputs are the parameters describing the successive segments as already discussed as well as parameters describing the noise. The outputs are the contour area, the actual contour in terms of the y coordinates corresponding to any x coordinate, and the coordinates of the extremity of the contour. The program is valid for both takeoff and landing trajectories. Since the equations are so simple, the program executes in seconds. For this reason, the program, when suitably implemented, would be ideally suited to on-line piloted simulation.

Another consideration concerns the accuracy of the closed-form solution. This accuracy has been investigated in Ref. 4 by comparing the results from the closed-form solution with that from the large computer contour program for identical conditions. Many examples for takeoff and landing trajectories of a conventional jet transport aircraft were used, too numerous to detail here. A typical comparison is shown in Fig. 3 by an example two-segment takeoff with thrust cutback in which ground attenuation and shielding have been omitted. Two sets of data are shown. First, the contour represents the results obtained from the program of Refs. 1 and 2 using the IBM-360 computer. The program is about 100,000 bytes in size, and a substantial amount of time and effort was required to obtain these results. Second, the crosses shown represent the results from the closed-form solution using the hand-held HP-67 calculator. Comparison shows excellent agreement. As

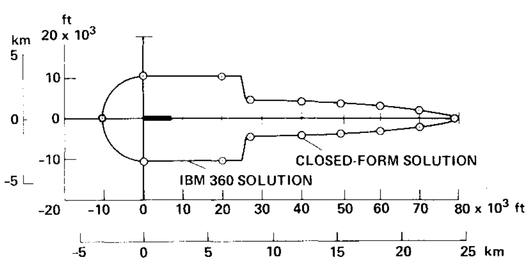

Fig. 3 Typical comparison of noise contours.

Table 1 Comparison of closed-form solution and computer program

Example	Area, km² Closed-form solution	IBM-360 program
1	154.93	154.9
2	154.93	155.4
3	100.49	99.5
4	17.59	17.4
5	24.22	24.1

for the contour areas, the results for several examples[4] are illustrated in Table 1 where it can be seen that the accuracy is within 1%.

Concluding Remarks

The simplicity of the noise analysis presented herein enables one to obtain the noise contour, its area, and its extremities for an arbitrarily complex flight path for both takeoffs and landings. The method is simple and fast, and results can be obtained either by manual calculation or by means of a small programmable calculator. The analysis reveals the fundamental nature of the contours and how the various factors that influence its size and shape enter into the analysis.

It should be noted that the effects of ground attenuation and shielding have been omitted from the analysis. Generally, their effects are important only on the initial portion of flight, and are highly dependent upon aircraft configuration. Preliminary analysis shows that such effects could be included, and further work in this direction might be warranted.

It is also worth emphasizing that the single-event contour discussed herein is the obvious choice for purposes of minimizing noise impact. The impact of multiple flights of the same type are handled by an obvious extension of the single-event results.

References

[1] "A Study of the Magnitude of Transportation Noise Generation and Potential Abatement, Vol. III—Airport/Aircraft System Noise," Serendipity, Inc., for the Department of Transportation, DOT OST-ONA-71-1, Nov. 1970.

[2] Safeer, H. B. and Williams, L. J., "Airport Noise Exposure Contour Model User Manual," Department of Transportation, DOT OST-ONA-72-3, Aug. 1972.

[3] "The FAA Integrated Noise Model Version 1, Basic User's Guide," Federal Aviation Administration, FAA-EQ-78-01, 1978.

[4] Stewart, E. C. and Carson, T. M., "A Closed-Form Solution for Noise Contours," NASA TP-1432, 1979.

Use of Similitude in Analyzing Aircraft Windshield Anti-Icing Performance

R. Ross*

Ross Aviation Associates, Sedgwick, Kan.

Nomenclature

f = function in Eq. (1)

P_A = ambient pressure, lb/ft²

P_B = bleed airflow pressure, lb/ft²

T_A = ambient temperature, °R

Presented as Paper 79-1822 at the AIAA Aircraft Systems and Technology Meeting, New York, N.Y., Aug. 20-22, 1979; submitted Dec. 6, 1979; revision received March 3, 1980.

Index categories: Analytical and Numerical Methods; Testing, Flight and Ground; Thermal Modeling and Analysis.

*Director. Associate Fellow AIAA.

T_B = bleed airflow temperature, °R
T_W = windshield surface temperature, °R
V = aircraft speed, ft/s
W_B = bleed airflow rate, lb/s
X = distance along windshield, ft

Introduction

SIMILITUDE can be defined as a property that permits the reduction of the number of parameters or variables upon which a physical phenomenon depends. Such a reduction has obvious desirable consequences, especially for the experimentalist. It means that a single test may result in obtaining all of the information normally requiring a whole series of tests. A prime example is the hypersonic similarity rule. Taking measurements on a slender body at high Mach number and the use of the hypersonic similarity rule results in data for a whole family of related thinner or fatter bodies at correspondingly different Mach numbers. Thus similitude can aid in planning an experiment as well as assist in analyzing the resulting data. The goal is to reduce the number of independent quantities in an experiment because that provides an important simplification and reduces the number of measurements required to define the solution.

Development of the Prediction Model

The first step in seeking similitude is to list the dependent and independent quantities in the problem. This step is by far the most important because the validity of the results depends on the correctness of the quantities chosen. The appropriate quantities in this problem are: bleed air mass flow, bleed air temperature, bleed air pressure, ambient temperature, ambient pressure, aircraft airspeed, position on the windshield, and surface temperature. At this step it is desirable to distinguish the dependent variable from the independent variables or quantities and to express the dependent variable, in this case the windshield surface temperature T_W, as a function of the independent variables.

The second step is the application of dimensional analysis which will provide a reduction in the number of independent quantities by combining them into a smaller number of dimensionless groups. These are frequently the familiar dimensionless numbers such as Reynolds number, Mach number, etc. Application of the familiar Buckingham π theorem will show the number of dimensionless products that exist in the problem, i.e., the number of dimensionless products is equal to the number of original quantities in the list reduced by the rank of the dimensional matrix formed for the fundamental units (mass, length, time, temperature) of the eight parameters previously listed. The rank of this matrix is four so that there should be four dimensionless products. The form of the unknown function relating these products is not revealed by dimensional analysis but it can be derived from analysis of observations.

The third step in the analysis is to bring in any supplementary information that might lead to a further reduction in the number of independent quantities. After the application of this step we are ready to try the results on the problem at hand. The pressure ratio, P_B/P_A, can be eliminated as a parameter by recalling that in incompressible flow, the absolute level of pressure is irrelevant, because pressure plays only a dynamic and not a thermodynamic role—only its gradient is significant. Thus the final relation is

$$T_W/T_A = f(W_B V/X^2 P_A, T_B/T_A) \tag{1}$$

Test Procedure

The tests were conducted on a Gates Learjet M35. The M35 is equipped with Garrett Air Research TFE 731 turbofan engines. The hot anti-icing air is obtained as bleed from the engines and is supplied to the windshield on the aircraft through a duct exiting at the center of the lower side of the windshield as shown in Fig. 1. Three thermocouples were attached to the windshield at the locations shown in Fig. 2. Test data were recorded for a variety of flight conditions as shown in Table 1.

Table 1 Flight test conditions

Condition	Range
Altitude, 1000 ft	1.8-45
Ambient temperature, °F	−90-61
Airspeed, knots	158-225
Bleed flow, lb/min	4-9
Bleed temperature, °F	250-330

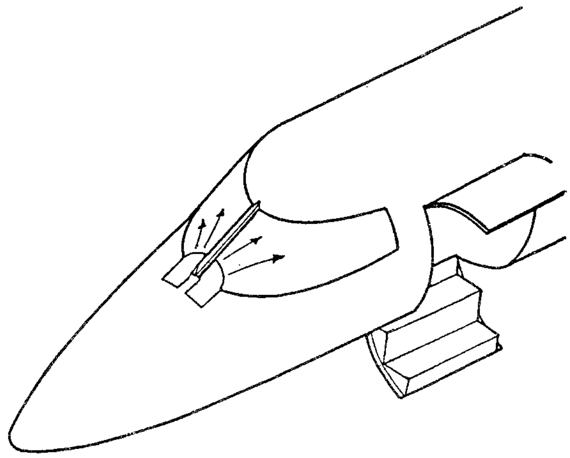

Fig. 1 Windshield anti-icing system.

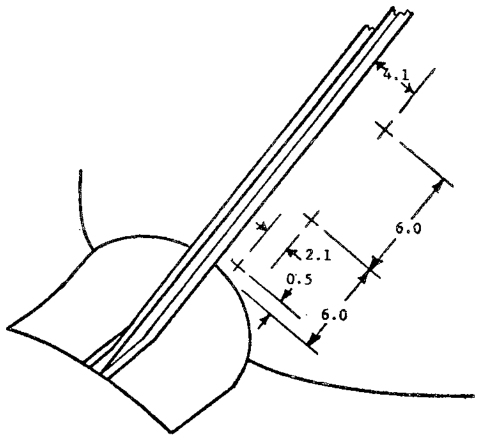

Fig. 2 Location of thermocouples.

Data Analysis

The test data were reduced to the three dimensionless groups as indicated by Eq. (1) and plotted on semilog graph paper. Inspection of the data indicated that there were sets of data points corresponding to nearly constant values of T_B/T_A. These were used as starting points for straight line curve fitting.

In order to determine a uniform family of curves fitting the data and conforming to Eq. (1), it was helpful to prepare additional working curves. A plot of T_W/T_A vs T_B/T_A for discrete values of $W_B V/X^2 P_A$ was prepared to find the variation for lower bleed temperatures and to give better definition to the shape of the curves. Linear regression analysis was applied to the points in this curve as they were

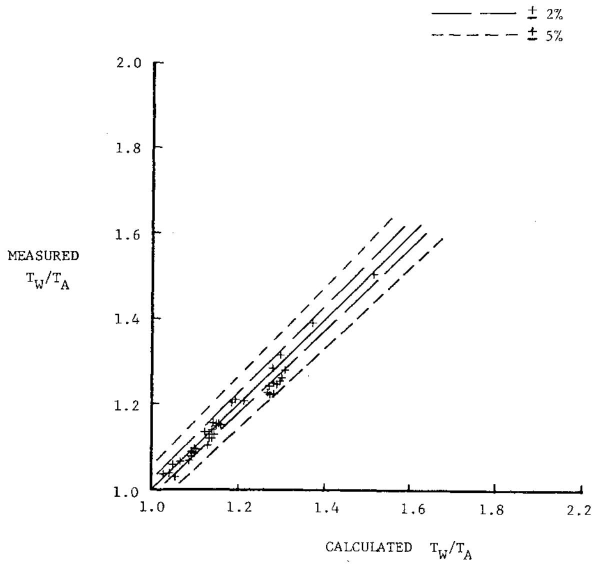

Fig. 3 Computational accuracy.

determined from the preliminary lines drawn through the data in the initial plot.

Further inspection of the data plots revealed the numerical equation that represents the function of Eq. (1). This is given by

$$T_W/T_A = 1.0 + [0.2508 + 0.0952 \log_{10}(W_B V/X^2 P_A)] \times (T_B/T_A - 1) \qquad (2)$$

Discussion of Results

The test of validity of such a technique as derived is to calculate the value of the windshield surface temperature based on known test conditions and compare that value with the corresponding measured value. This was done for all of the available test data. The results are shown as measured vs calculated T_W/T_A in Fig. 3. Error bands of ± 2 and $\pm 5\%$ are shown and the accuracy is satisfactory. On the basis of 45 test points the average difference between measured and predicted T_W/T_A is 0.016.

Conclusions

The formulation of a prediction technique has been accomplished with the use of similitude. The method allows the prediction of windshield surface temperature, assuming that proper information about the aircraft conditions, weather conditions, and hot bleed air conditions is known. The following conclusions are reached:

1) A dimensional analysis technique can successfully express windshield temperatures in the form of a mathematical function whose mathematical equation is unknown.

2) Experimental data from aircraft flight tests are required to determine the form of the correlation leading to the prediction technique.

3) With the appropriate aircraft and atmospheric information available, windshield surface temperatures can be predicted with acceptable accuracy.

4) Given sufficient experimental data, the exact mathematical equation representing the problem under investigation can be derived.

Acknowledgment

The author expresses appreciation to The Gates Learjet Corporation for its cooperation in releasing the data for publication.